

Thermal emission from layered structures containing a negative-zero-positive index metamaterialLi-Gang Wang,^{1,2} Gao-Xiang Li,^{1,3} and Shi-Yao Zhu^{1,2,4}¹Centre of Optical Sciences and Department of Physics, The Chinese University of Hong Kong, Shatin, N. T., Hong Kong, China²Department of Physics, Zhejiang University, Hangzhou 310027, China³Department of Physics, Huazhong Normal University, Wuhan 430079, China⁴Department of Physics, Hong Kong Baptist University, Kowloon Tong, Hong Kong, China

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Motivated by the realization of the optical Dirac dispersion in the homogenous negative-zero-positive index (NZPI) material, we make a theoretical investigation on the properties of thermal emission in layered structures containing the NZPI medium. We find that when the thermal emission frequency is close to the Dirac point of the NZPI medium, the spectral hemispherical power of thermal emission in such a structure is strongly suppressed and the emission can become a high directional source with large spatial coherence.

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In the last decade, the subject on modifying thermal radiation has attracted great attention^{1–5} due to its potential applications in thermally pumped optical devices, such as tunable infrared emitters, thermophotovoltaic energy conversion devices,⁶ or solar absorbers/reflectors. It is well known that a perfect thermal emitter follows Planck's law of blackbody radiation. However, the thermal emission spectra for realistic structures do not completely follow the Planck's law but strongly depend on the material dispersions and their structures. It is widely recognized that artificial structures such as multilayered periodic structures, two-dimensional or three-dimensional photonic crystals can greatly enhance or suppress the thermal radiation.^{2,7–10} Using these properties, one can design narrow-band thermal emitters^{11–13} with the ability to select the band frequency, directional or polarization of the emission, and design wide-band thermal emitters exhibiting near-blackbody thermal emission within a certain wavelength range,^{1,14–16} and also design wide-angle absorbers/reflectors.¹⁷

Recently, schemes for various emitters/absorbers based on metamaterials were proposed,^{18–21} with some specific properties such as wavelength-selective wide-angle absorbers²⁰ and narrow-band angle-insensitive thermal emitters.²¹ The emission or absorption properties have been extensively studied on the structures containing the so-called negative-index metamaterials.²² On the other hand, it has been found that in certain two-dimensional photonic crystals there is a Dirac-type dispersion for electromagnetic field within a particular frequency region, and such a Dirac dispersion leads to some unique properties for the optical fields such as conical diffraction,²³ a “pseudodiffusive” scaling,²⁴ and the photon's Zitterbewegung.²⁵ The Dirac dispersion can also be exhibited in a homogenous negative-zero-positive index (NZPI) medium.^{26,27} Since the density of the electronic states near the Dirac point of Graphene is unique and tends to zero,²⁸ it is expected that the density of the photon's states is also gradually close to zero near the Dirac point of the NZPI medium. In this paper, we will consider the properties of thermal emission from a layered structure containing the NZPI metamaterial around the Dirac point. We expect that the spectral hemispherical power of thermal emission in such structures is strongly suppressed when the emission frequencies are close to the Dirac point of the NZPI medium. More

interestingly, our analysis reveals that, as the frequency approaches the Dirac point, the emission becomes a highly directional source with large spatial coherence.

Consider the layered structure containing the NZPI metamaterial, as depicted in Fig. 1. For simplicity, we assume that the substrate is metal and the nonmagnetic emitter layer is with thickness d_2 and the parameters $\epsilon_2=1.96+i0.14$ and $\mu_2=1$, which corresponds to the complex refractive index $n_e \approx 1.4+i0.05$. The upper layer is the NZPI metamaterial with thickness d_3 . For simplicity, we take the Drude model for both the relative permittivity and permeability of the NZPI metamaterial^{26,29}

$$\epsilon_3(\omega) = 1 - \frac{\omega_{ep}^2}{(\omega^2 + i\gamma\omega)}, \quad (1)$$

$$\mu_3(\omega) = 1 - \frac{\omega_{mp}^2}{(\omega^2 + i\gamma\omega)}, \quad (2)$$

where ω_{ep} and ω_{mp} are the controllable electronic and magnetic plasma frequencies, respectively; and γ is related to the absorption of the NZPI medium. Here we assume $\gamma \ll \omega_{ep,mp}$ for the very weak absorption cases. In the previous investigation,²⁶ we have verified that when $\omega_{ep}=\omega_{mp} \equiv \omega_D$ and $\gamma=0$ (no loss), then the dispersion of the material satisfies a linear dispersion: $k_3(\omega)=(\omega-\omega_D)/v_D$, where v_D is the group velocity of light near frequency ω_D inside the NZPI medium with the wave number $k_3(\omega)$. Such a linear dispersion with a nonzero ω_D is defined as Dirac dispersion and the upper and lower passbands of light passing through the bulk NZPI medium touch together at frequency ω_D , therefore the frequency ω_D is called as the Dirac point of the bulk NZPI metamaterial. In our calculation we take $\epsilon_1=-12.0$ for the metal substrate and thickness $d_2=50\lambda_D$ (here the wavelength λ_D is corresponding to ω_D).

First, let us investigate the spectral hemispherical power from such structures as shown in Fig. 1. Using Kirchoff's second law³⁰ which relates emittance directly to absorbance, the thermal emission intensity within the frequency interval $\omega-(\omega+d\omega)$ is

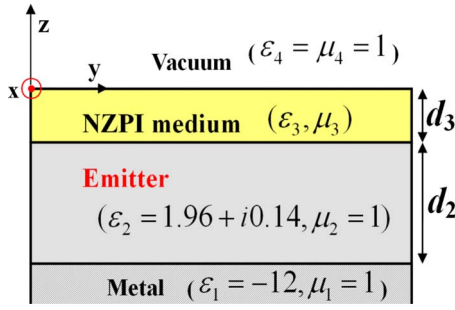


FIG. 1. (Color online) Schematic of the layered structure with a NZPI medium.

$$I(\omega, \theta, \phi, T) = \tau(\omega, \theta, \phi)\Theta(\omega, T), \quad (3)$$

where $\tau(\omega, \theta, \phi)$ is the emittance of the layered structure at frequency ω , $\Theta(\omega, T) = \hbar\omega^3 / [4\pi^3 c^2 (e^{\hbar\omega/k_B T} - 1)]$ is the function for the blackbody radiation intensity, θ is the angle between the emitted thermal light ray and the normal line (along the z axis), and ϕ is the projected angle of the emitted ray on the x - y plane. From Eq. (3), we can obtain the spectral hemispherical power¹⁰

$$U(\omega, T) = \int_{2\pi} I(\omega, \theta, \phi, T) \cos \theta d\Omega, \quad (4)$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \sin \theta \cos \theta \tau(\omega, \theta, \phi)\Theta(\omega, T). \quad (5)$$

Here $d\Omega$ is the differential solid angle in which thermal radiation is emitted, and we would like to mention that, since the substrate is metal, there is no need to calculate the transmission. Based on the transfer-matrix method,³¹ we can readily obtain the emittance $\tau(\omega, \theta, \phi) = 1 - R(\omega, \theta, \phi)$, where $R(\omega, \theta, \phi)$ is the reflectivity from the structure as shown in Fig. 1. In this paper, we only consider that the emitted light fields are TE polarized and similar results can be obtained for the TM-polarized thermal emission.

Figure 2 shows the spectral hemispherical power $U(\omega, T)$ as a function of frequency for different situations. We choose to align the frequency of the Dirac point, $\omega = \omega_D$, with the

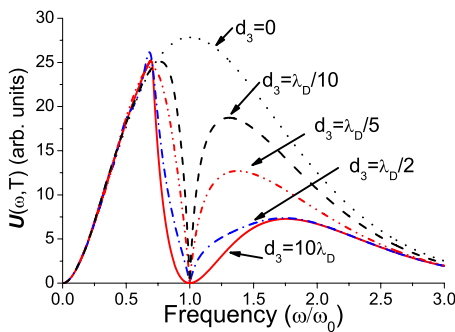


FIG. 2. (Color online) The dependence of the spectral hemispherical power on the frequency under different values of thickness d_3 . Other parameters are $\omega_D/2\pi = 10$ GHz and $\gamma = 0.00001$ GHz.

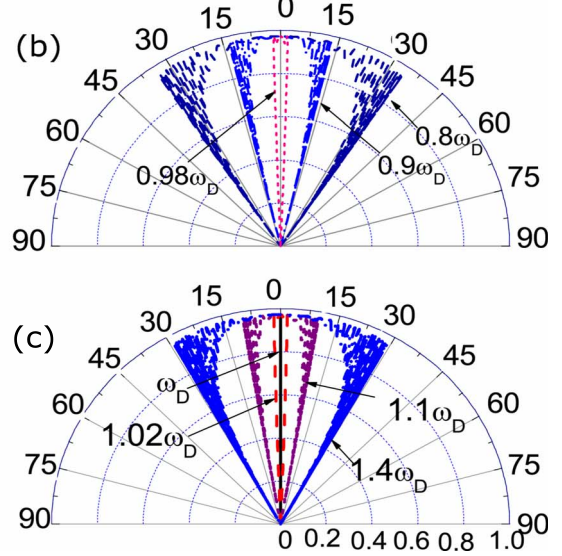
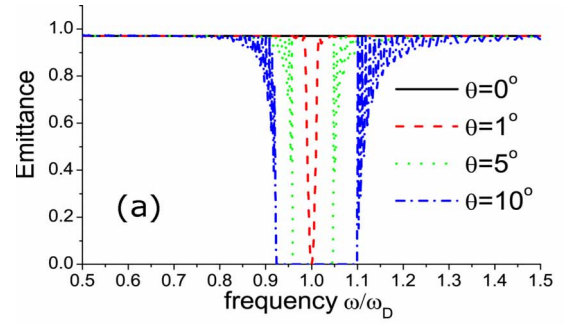


FIG. 3. (Color online) (a) The emittance as a function of frequency at different angles θ and [(b) and (c)] the angular dependence of the emittance at different frequencies labeling in the figure, with $d_3 = 10\lambda_D$.

maximum of the Planck blackbody radiation for the structure without the NZPI medium, i.e., the dot line for $d_3 = 0$. In the presence of the NZPI medium, it is clear that the spectral hemispherical power is strongly suppressed as ω approaches to the Dirac point. This reflects the extremely low photonic density of states near the Dirac point. In Fig. 2, we can see that as the thickness of the NZPI medium increases, near the Dirac point the spectral hemispherical power is suppressed more and more, and quickly approaches the limit of the structure with the bulk NZPI medium because the thicker NZPI film reduces the interface effect and manifests the perfect Dirac spectrum.²⁶ In principle, for the structure with the infinite thick NZPI medium (i.e., $d_3 \rightarrow \infty$), the thermal emission can only emit photons along a single direction (normal line) at Dirac point.

Figure 3(a) shows the emittance as a function of frequency along different angles. It is seen that along the normal line ($\theta = 0$), the emittance at all frequencies is almost a constant while with the increasing of the emitted angle the emittance is strongly suppressed near the Dirac point. At the inclined angles, the emittance is similar to that in the photonic band-gap structures (for example, see Refs. 1 and 2). From Figs. 3(b) and 3(c), one can find that when the frequency of the emitted light is close to the Dirac point, the angular dependence of thermal emission is sharply centered

with zero degree (i.e., the normal line). For example, the curve for $\omega=0.98\omega_D$ in Fig. 3(b) and the curve for $\omega=1.02\omega_D$ in Fig. 3(c) have a sharp angular distribution around zero degree; especially, when $\omega=\omega_D$, the emission has an extremely sharp angular distribution (almost becomes a single line), see in Fig. 3(c).

Now let us turn to study the coherence properties of the thermal emission from such structures containing the NZPI medium. Recent studies have shown that the coherence of a thermal radiation can be drastically different from that of the blackbody radiation in various systems such as photonic crystals^{1,3,9} and microcavities.³² Following the method used in Refs. 33 and 34, we can analytically obtain the output two-point cross-spectral density tensor for the TE-polarization thermal emission in our system as follows:

$$\begin{aligned} W(\mathbf{r}_1, \mathbf{r}_2; \omega) = & \omega \mu_0 \varepsilon_2'' \Theta(\omega, T) \int_0^{d_2} dz_0 \\ & \times \int_0^\infty dk_{\parallel} \frac{k_{\parallel}}{\beta_2} |\exp[i\beta_4 z]|^2 \\ & \times \left(|F_+|^2 \left[\frac{J_1(k_{\parallel} x)}{k_{\parallel} x} \hat{x} \hat{x} \right. \right. \\ & \left. \left. + \left[J_0(k_{\parallel} x) - \frac{J_1(k_{\parallel} x)}{k_{\parallel} x} \right] \hat{y} \hat{y} \right] \right), \quad (6) \end{aligned}$$

where

$$\begin{aligned} F_+ = & \frac{t_{2/4} [e^{i\beta_2(d_2-z_0)} + r_{21} e^{i\beta_2(d_2+z_0)}]}{(1 - r_{21} r_{2+} e^{i2\beta_2 d_2})}, \\ r_{2+} = & \frac{r_{23} + r_{34} e^{i2\beta_3 d_3}}{1 + r_{23} r_{34} e^{i2\beta_3 d_3}}, \\ t_{2/4} = & \frac{(1 + r_{23})(1 + r_{34}) e^{i\beta_3 d_3}}{1 + r_{23} r_{34} e^{i2\beta_3 d_3}}. \end{aligned}$$

Here the points $\mathbf{r}_1=(0,0,z)$ and $\mathbf{r}_2=(x,0,z)$ are in the vacuum. The parameter k_{\parallel} is the magnitude of the transversal wave vector $\mathbf{k}_{\parallel}=(k_x, k_y)$ along the interfaces of the layers. The β_j ($j=1,2,3,4$) are the magnitude of the z component wave vector in the j th layer (see Fig. 1). If $\text{Re}(k_{\parallel}) < \text{Re}(k_0 \sqrt{\varepsilon_j \mu_j})$, that is, the corresponding wave in the j th layer is a propagating mode, then β_j is expressed as $\beta_j = \sqrt{\varepsilon_j \mu_j [k_0^2 - k_{\parallel}^2 / (\varepsilon_j \mu_j)]}^{1/2}$, where $k_0 = \omega/c$. On the other hand, if $\text{Re}(k_{\parallel}) > \text{Re}(k_0 \sqrt{\varepsilon_j \mu_j})$, the field corresponds to an evanescent mode in the j th layer, and then $\beta_j = i[k_{\parallel}^2 - \varepsilon_j \mu_j k_0^2]^{1/2}$. The reflection coefficients for the TE wave at the single interface i - j are defined as

$$r_{ij} = \frac{\mu_j \beta_i - \mu_i \beta_j}{\mu_j \beta_i + \mu_i \beta_j}. \quad (7)$$

In Figs. 4(a) and 4(b), we show the components W_{xx} and W_{yy} of the normalized cross-spectral density tensor for the thermal emission from the structure with the NZPI medium.

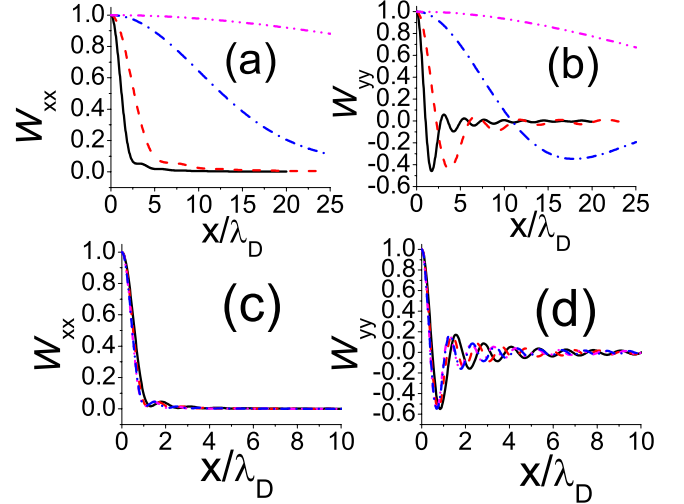


FIG. 4. (Color online) The values of [(a) and (c)] W_{xx} and [(b) and (d)] W_{yy} as functions of distance x . [(a) and (b)] The structures containing the NZPI metamaterial with $d_3=10\lambda_D$ and [(c) and (d)] the case without the NZPI medium (i.e., $d_3=0$), where $\omega/2\pi=8$ GHz (solid curves), 9 GHz (dashed curves), 9.8 GHz (dashed-dot curves), and 10 GHz (dashed-dot-dot curves). Note that different curves are nearly overlapped in (c) and (d).

Here we choose the thickness of the NZPI medium $d_3=10\lambda_D$ and in this case the NZPI medium is sufficient to show the Dirac dispersion. It is clear seen that as the frequency approaches to the Dirac point of the NZPI medium, the emission has a larger length of transversal spatial coherence. From Figs. 4(a) and 4(b), one can find that, near the Dirac point, the thermal emission becomes a coherent source with a transversal coherent length more than ten wavelengths in the present case. The reason is that we can see from Fig. 3 and Eq. (6) near the Dirac point of the NZPI medium, the propagating modes emitted by the emitter which can propagate out of the medium own very low magnitude of the transversal wave vectors, the density of these propagating modes is greatly reduced due to the modification of the Dirac dispersion and there exists only a very few of photon's states allowed to be emitted out from the emitter. Therefore a coherent source from the thermal radiation can be obtained near the Dirac point. Meanwhile, from Figs. 4(a) and 4(b), we can also find that as the frequency of the emitted light is away from the Dirac point, the density of the propagating modes emitted by the emitter increases and the magnitude of the transversal wave vectors of these propagating modes varies a wide range, the transversal coherent length quickly decreases and is nearly close to one wavelength, which is in agreement with the blackbody radiation and is also similar to the situations in Figs. 4(c) and 4(d) for the structure without the NZPI medium.

In summary, we have analyzed thermal emission in layered structures containing the NZPI metamaterials with the Dirac point. It shows that the spectral hemispherical power of thermal emission is strongly suppressed when the frequencies are close to the Dirac point of the NZPI medium. As the

thermal emission frequency closes the Dirac point, the emission from the structure becomes a light source of highly direction and large spatial coherence. Our results will lead to the potential applications in integral optics and optical-based devices, such as the coherent narrow-angular thermal emitters.

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